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Finite element analysis of trapezoidal cross section lossy waveguide

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Abstract Rectangular waveguide having trapezoidal cross section filled with lossy medium is analysed using Finite Element Method (FEM). The effect of lossy medium on TE_{10}, TE_{20} mode propagation constant and on bandwidth of trapezoidal cross section waveguide is worked out

Keywords Finite element analysis, rectangular waveguide, lossy medium,

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The dispersion characteristics of arbitrary shaped waveguide with sharp metal edges are found using finite element method by Webb [1]. Kalamse [2] has used the finite element method for analysing the rectangular and triangular patch microstrip resonator with shear and shape deformation respectively. Chaudhari [3] used the same method for analysis of rectangular waveguide with deformation at vertical sides.

In this paper, we have analysed the trapezoidal cross section lossy waveguide by considering the shape deformation on vertical side as shown in Figure 1. The dimension of waveguide is considered as $a = 2t$ and $b = t$ to make the problem dimensionless. The deformation

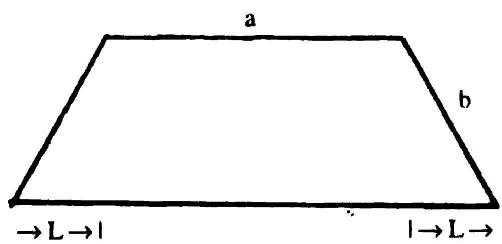

Figure 1. Trapezoidal cross section of waveguide.

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length L is increased with l/8. For each deformation length L, the eigenvalues and eigenvectors are obtained. The TE_{10} and TE_{20} mode are identified using field plotting. The variation of attenuation constant, phase constant with frequency is shown graphically for each deformation length L.

We consider a cross section of lossy wave guide having trapezoidal shape (Figure 1), whose relative permittivity tensor $[\varepsilon]$ is

$$[\varepsilon]=\begin{bmatrix}\varepsilon_x & 0 & 0\\ 0 & \varepsilon_y & 0\\ 0 & 0 & \varepsilon_z\end{bmatrix}, \tag{1}$$

$$\;_i - j\varepsilon_i, \qquad i = x, y, z. \tag{2}$$

With a time-dependence form $\exp(j\omega t)$ being implied, Maxwell's equation gives the following wave equation :

$$\nabla x\,([\varepsilon]^{-1}\,\nabla \times \mathrm{H}) = K_o^2\,H, \tag{3}$$

where $K_o = \omega^2\mu_0\varepsilon_o$

Here, ω is the angular frequency, ε_0 and μ_0 are the permittivity and permeability of the free space, respectively.

Dividing the cross section Ω of the guide into a number of first-order square finite-elements, the magnetic fields within each element are defined in terms of those at the corner and midside nodal points ;

$$H = [N]^T\,\{H\}_e\,\exp(-\gamma z) \tag{4}$$

where

$$[N]=\begin{bmatrix}\{N\} & \{0\} & \{0\}\\ \{0\} & \{N\} & \{0\}\\ \{0\} & \{0\} & j\{N\}\end{bmatrix}, \tag{5}$$

$$\{N\}^T = [N1\ N2\ N3\ N4], \tag{6}$$

and $$\{H\}_e^T = [\{Hx\}_e \{Hy\}_e\ \{Hz\}_e]. \tag{7}$$

Here $\{N\}$ is the shape function vector ; $\{0\}$ is a null vector, T, $\{\ .\ \}$, and $\{\ .\ \}^T$ denote a transposition, a column vector, and a row vector, respectively ; and $\{Hx\}_e$, $(Hy\}_e$, and $(Hz\}_e$ are complex magnetic-field vectors corresponding to the nodal points within each element e.

Application of the standard finite-element technique *via* a Galerkin procedure to (3) gives the following global matrix equation :

$$[S]\,\{H\} + K_0\,[T']\,\{H\} - Ko^2\,[T]\,\{H\} = 0. \tag{8}$$

By introducing the divergence condition and rearranging the above equation for γ^2 $(-\lambda)$ [4], we get

$$\lambda^2[A]\,\{Ht\} + \lambda[B]\,\{Ht\} + [C]\,\{Ht\} = 0. \tag{9}$$

Since (9) is a complex quadratic eigenvalue problem, it can be reduced to the following standard from :

$$\begin{bmatrix} [0] & [U] \\ -[A]^{-1}[C] & -[A]^{-1}[B] \end{bmatrix} \begin{bmatrix} \{Ht\} \\ \{\bar{H}t\} \end{bmatrix} = \lambda \begin{bmatrix} \{Ht\} \\ \{\bar{H}t\} \end{bmatrix}, \tag{10}$$

where $\{\bar{H}t\} = \lambda\{Ht\}$. (11)

Although {10} shows a little complexity in comparison with other formulations [5, 6], it is a standard eigenvalue problem whose eigenvalue directly corresponds to the propagation constant γ. Thus, one can avoid unnecessary interations using complex frequencies. The only disadvantage of this form is that it involves $4N_p$ unknown components in each eigenvector compared with $2N_p$ components in the original system, where N_p is the number of nodal points.

The deformation length L is increased with t/8. For each deformation length L, the eigenvalues and eigenvectors are obtained. The TE_{10} and TE_{20} mode are identified using field plotting.

The graphs for k_0t *Vs alpha/*k_0 and *beta/*k_0 for TE10 and TE20 modes are plotted as shown in sample Figure 2(a, b). From these graphs for different deformed length L, we have calculated the cut-off frequencies for TE10 and TE20 mode. The cut-off frequencies are calculated from the graphs of *alpha/*k_0 *Vs* k_0t, for minimum attenuation [7]. From these cut-off frequencies, bandwidth is calculated. For each value of L this procedure is repeated.

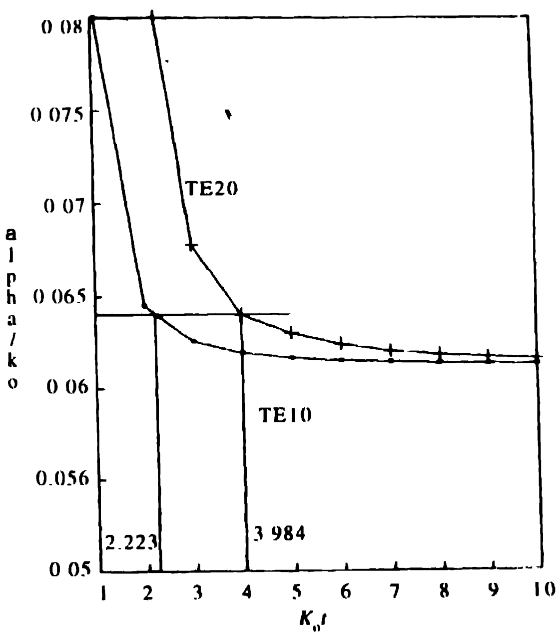

Figure 2(a). Variation of TE_{10} and TE_{20} mode alpha due to deformation L = 0 0

The following trend of frequency change is observed for nonlossy [3] and lossy medium (Table. 1), for increase in deformation length L with the step of t/8 :

(i) the value of TE10 cut-off frequency increases.

(ii) the value of TE20 cut-off frequency increases upto $2t/8$ and then decreases at $3t/8$, and then again it increases.

(iii) the band width increases upto deformed length $L = 2t/8$ and then decreases at $3t/8$ and again increases with increase in deformation length.

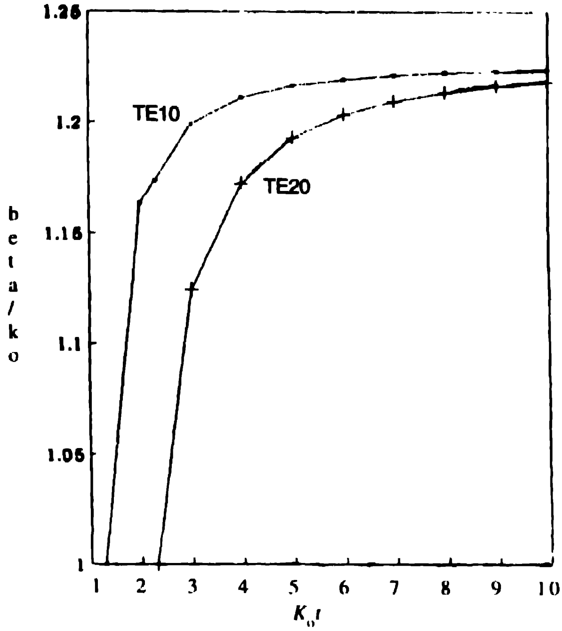

Figure 2(b) Variation of TE_{10} and TE_{20} mode beta due to deformation L = 0 0

Due to consideration of attenuation on the wave, values calculated in lossy medium are more practical than air medium. Also due to lossy medium, lower cut-off value is increased and second cut-off value is increased more. So that bandwidth is found more in this case.

Table 1 Variation in Bandwidth due to lossy medium and deformation at the top in inward direction with the step of L = t/8

Sr No.	Deformation L	Cut-off K_0 TE20	Cut-off K_0 TE10	cut-off f in GHz TE20	cut-off f in GHz TE10	B.W in GHz
1	0	3.9845	2 2234	19 0221	10 6145	8.4076
2	t/8	4 3865	2.4446	20 9412	11.6772	9.2639
3	2t/8	4.7245	2.2634	22 5549	12 5241	10 0307
4	3t/8	3 9877	2 8454	19 0373	13.5840	5.4533
5	4t/8	4 5846	3.3564	21.8870	16.0235	5.8634
6	5t/8	5 1545	3 8546	24 6077	18 4019	6 2058
7	6t/8	5 6645	4 2946	27 0242	20 5025	6.5398

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